Programming and Classification 2022

List 1

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I. When the numbers start to be really big ...

1. Which number is greater - the number of atoms in the Universe or the number of all binary strings of length 500?
2. To sort n-elements array our implementations of `InsertSort` and `QuickSort` algorithms need n^2 and $2n \log n$ comparisons, respectively. Each comparison takes $1.2 \quad ns$ [1]. What time is necessary if you need to sort an array with $n = 1.000$, $1.000.0000$,$200.000.000$ elements using both algorithms? What size of arrays you can sort using both algorithms in one day?
3. Draw graphs of functions $f_1(n) = n$, $f_2(n) = n^2$, $f_3(n) = \log n$ for $n \in [1, 30000]$ on a single figure. Similarly, draw functions $g_1(n) = 50n^4$ and $g_2(n) = 2^n$ for $n \in [1, 40]$.

II. Some important functions ...

1. Compute Euclidean distance between vectors $[1, 2, 3]$ and $[5, 3, 4]$.
2. Compute Hamming distance between vectors $[1, 1, 1, 0, 1]$ and $[0, 0, 1, 1, 0]$.
3. Compute cosine value of the angle between vectors $[1, 2, 1]$ and $[1, -1, 0]$.
4. Recall Stirling's formula for $n!$. Approximate $\binom{2n}{n}$.

III. Probability and rare events ...

1. We choose independently at random three points i, j, k from the set $\{-1, 0, 1\}$. Let D be the length of the vector (i, j, k). Compute the expected value and variance of D.
2. We throw independently $n > 3$ balls into $m > 2$ buckets at random[2]. Balls are thrown independently.
   (a) What is the probability that the i-th and the j-th ball are placed in a different bin?
   (b) What is the expected number of balls in the first bin?
   (c) Estimate the probability that there is a bin with $m/2$ balls (☠).
3. For security reasons Roman and Bogumił are obliged to reset their passwords every Monday. Passwords should be generated as 4 lower case random letters stroked in a standard qwerty keyboard. System administrators noticed that in a years 2011-2018 they had the same password in exactly 10 weeks. Do they fulfil the security policy rules?
4. Rudolf is obliged to follow the same security policy. System administrators noticed that in 2018 Rudolf had Polish word `zupa` during two weeks in May and one week in October. He claims that this word has been randomly generated three times. Shall we believe him ?

[1] $1ns$ is a *nanosecond* $= 10^{-9}s$.

[2] In English *at random* means **uniformly at random**. In Polish is different, you have to precise the probability distribution.